

Dimostrazione. Siano

$$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \quad \text{e} \quad B = \begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix}$$

Allora

$$AB = \begin{pmatrix} a\alpha + b\gamma & a\beta + b\delta \\ c\alpha + d\gamma & c\beta + d\delta \end{pmatrix}$$

mentre per $z \in \mathbb{C}$ abbiamo

$$\begin{aligned} f_A \circ f_B(z) &= f_A\left(\frac{\alpha z + \beta}{\gamma z + \delta}\right) = \frac{a \frac{\alpha z + \beta}{\gamma z + \delta} + b}{c \frac{\alpha z + \beta}{\gamma z + \delta} + d} \\ &= \frac{a(\alpha z + \beta) + b(\gamma z + \delta)}{c(\alpha z + \beta) + d(\gamma z + \delta)} = \frac{(a\alpha + b\gamma)z + a\beta + b\delta}{(c\alpha + d\gamma)z + c\beta + d\delta} = f_{AB}(z) \end{aligned}$$